From a Science Egg to a Science of Diagrams

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Abstract. In the last decade of his life, Peirce developed phaneroscopy and existential graphs as the basis for a proof of pragmaticism. To publish the proof, he wrote a series of articles for the *Monist*. The first two began with phaneroscopy. But in 1906, he added a version of tinctured existential graphs to the third article, *An Apology for Pragmaticism*. In 1908, he began a fourth article, which he never finished. One reason he stopped may be his remark in 1909: "Phaneroscopy, still in the condition of a science-egg, hardly any details of it being as yet distinguishable." Other reasons involve issues about the graphs, which he resolved in 1911. Although Peirce did not complete the proof, his writings inspired aspects of Lady Welby's *significs*, Wittgenstein's language games, and patterns of diagrams in every branch of science and engineering. Today, Peirce's theories of phaneroscopy and diagrammatic reasoning clarify critical issues in cognitive science. Among them are the methods of reasoning in linguistics, neuroscience and artificial intelligence.

1. Developments from 1903 to 1913

For Peirce, 1902 brought an end to two major projects: Baldwin's dictionary was finished, and funding for his *Minute Logic* was rejected. But three events in 1903 led him to rethink every aspect of his life's work: his Harvard lectures in the spring, his Lowell lectures in the fall, and his correspondence with Victoria Welby. As a guide to the new developments, the tree in Figure 1 shows his classification of the sciences and dependencies among them. Branches show the classification, and dotted lines show the dependencies. Sciences to the right of each dotted line depend on sciences to the left. Pure mathematics stands alone, and all other sciences and engineering depend on mathematics (CP 1.180ff, 1903).

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In summary, phaneroscopy depends on mathematics, which includes existential graphs as a formal logic. But as a diagrammatic logic, EGs can be used in two ways. For phaneroscopy, the option of changing shape is important. Nodes of a graph may be moved to match the shape of the image they represent. For logic, however, changing the shape does not change the meaning. Since the same notation can serve both purposes, EGs support Peirce's prediction that phaneroscopy "surely will in the future become a strong and beneficient science" (R645, 1909).

2. The Role of Diagrams in Phaneroscopy

For the third *Monist* article, *Prolegomena to an Apology for Pragmaticism*, Peirce chose a title that echoes Kant's *Prolegomena*. In it, he addressed Kant's three "transcendental questions": How is pure mathematics possible? How is pure natural science possible? How is metaphysics in general possible? The dotted lines in Figure 1 suggested the answer shown in Figure 2: diagrams, such as EGs, are mathematical structures that relate phaneroscopy, metaphysics, and the natural sciences to methods for thinking, talking, and acting in and on the world.

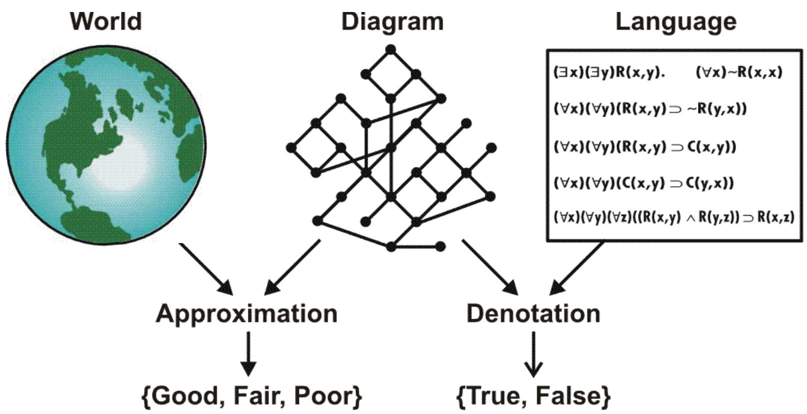

Figure 2: Diagrams relate thought and language to the world

The first sentence sets the stage: "Come on, my Reader, and let us construct a diagram to illustrate the general course of thought; I mean a System of diagrammatization by means of which any course of thought can be represented with exactitude" (CP 4:530). Figure 2 shows an important step beyond Tarski's model theory. Instead of a one-step mapping from the world to language, the diagram splits the mapping in two distinct steps.

Phaneroscopy maps some aspect of the world to a diagram, which is "an icon of a set of rationally related objects" (R293, NEM 4:316). It serves as a Tarski-style model for determining the denotation of languages, formal or informal. But when a continuous world is mapped to a discrete diagram, an enormous amount of detail is lost. Although the right side can be a precise map from a graph to a formal logic, it may be an approximate mapping from an informal diagram to the informal languages that people speak. In his career as a scientist, engineer, linguist, lexicographer, and philosopher, Peirce understood the complexity of both sides.

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With the new terminology, Peirce renamed the existential graph *sheet of assertion* as a *phemic sheet*, which "iconizes the Universe of Discourse [UoD], since it more immediately represents a field of Thought, or Mental Experience, which is itself directed to the Universe of Discourse, and considered as a sign, denotes that Universe" (R300). Figure 3 shows a phemic sheet derived by perception of the world and action on it.

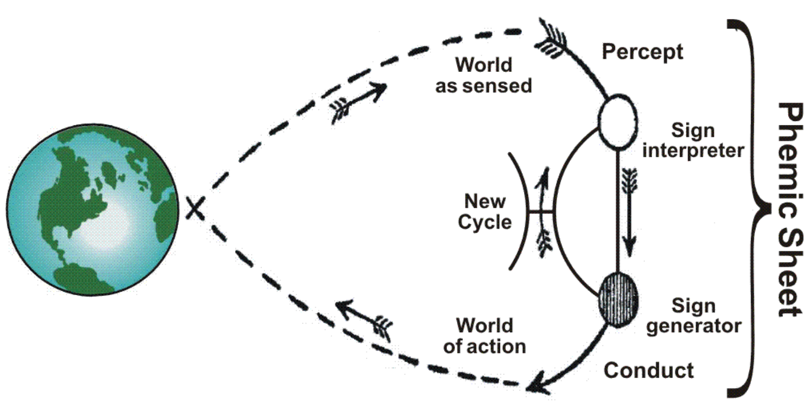

Figure 3: Deriving a phemic sheet by perception and action

Figure 3, adapted from a drawing by Uexküll (1920), shows how an animal of any species could sense and act upon the world. The "Mental Experience" (*Innenwelt*) of the animal is represented by a phemic sheet. The sign interpreter receives percepts (semes) from any external source or any organ in the body.

A simple stimulus-response would take milliseconds to relate a sensory seme to a seme that triggers an action. But repeated cycles would relate and combine semes and phemes for diagrammatic reasoning. A delome would be a sequence of phemes that answers a question or resolves a dispute.

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An appropriate logic should facilitate a proof of pragmaticism. Peirce stated the requirements in his *Prolegomena*: "a System of diagrammatization by means of which any course of thought can be represented with exactitude." Then "operations upon diagrams, whether external or imaginary, take the place of the experiments upon real things that one performs in chemical and physical research." The system has four aspects: (1) diagrams in EGs or other notations; (2) grammars for mapping languages to and from diagrams; (3) critic for evaluating the denotation {true,false} of diagrams in terms of a formal logic; and (4) perception and action for relating the world to the diagram. The arrows in the hexagon of Figure 4 indicate the flow of any course of thought.

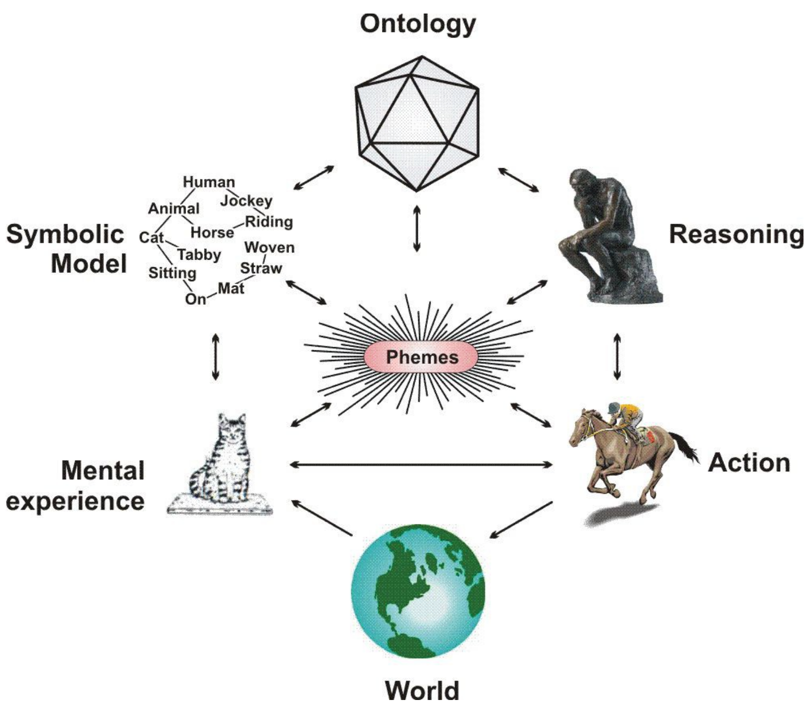

Figure 4: The flow of thought in an intelligent system

The hexagon in Figure 4 shows details implicit in Figure 2. The upper three corners and the starburst of phemes represent intelligent processing. The lower three corners correspond to the drawing by Uexküll in Figure 3. The arrow from mental experience to and from action supports routine habits or emergency responses. Behavior that requires complex reasoning may involve all the nodes and arrows.

As Peirce insisted, a diagram of information flow, such as Figure 4, is not a psychological theory. It may represent data that controls a robot or the thought of an alien being in a distant galaxy. But the word *exactitude* for representing "any course of thought" poses a challenge. As Figure 2 shows, the mapping between the world and a diagram can only be approximate, and the mapping between a diagram and a language can only be exact for notations that are designed to represent those diagrams. Approximations must be recognized and accommodated.

With his constant questioning, Peirce's ideas kept evolving. In 1907, he had stated the basis for his proof: "the Graphs break to pieces all the really serious barriers, not only to the logical analysis of thought but also to the digestion of a different lesson by rendering literally visible before one's very eyes the operation of thinking *in actu*" (CP 4.6, R298). 1909, he expressed his concerns about phaneroscopy "still in the condition of a science-egg" (R645). But in In the next two years, he addressed those issues and generalized existential graphs to accommodate them.

3. Relating Images to Diagrams

Since the semes and phemes that flow along the arrows of Figure 4 may contain uninterpreted percepts and images, ordinary existential graphs cannot represent them. In the letter L231, in which Peirce specified his most general notation for EGs, he mentioned his hopes of representing"stereoscopic moving images." To accommodate them, Sowa (2016, 2018) proposed *generalized* existential graphs (GEGs). Figure 5 shows Euclid's Proposition 1 stated in three kinds of GEGs: "On a given finite straight line, to draw an equilateral triangle."

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For details, see Sowa (2018) Reasoning with diagrams and images, *Journal of Applied Logics* **5:5**, 987-1059. http://www.collegepublications.co.uk/downloads/ifcolog00025.pdf

4. Significs

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During the following decade, correspondence between Peirce and Welby strongly influenced both. In 1903, Peirce had adopted Kant's abstract phenomenology. But in 1904, he coined the new word phaneroscopy, which he discussed in terms that were closer to Welby's emphasis on observation and mental experience. In his letters to her, Peirce added examples that clarified the motivation and explained the details of his abstract analysis. His classification of the sciences in 1903 (Figure 1) illustrates the differences, Peirce had sharply distinguished mathematics, phaneroscopy, and the normative sciences. With her emphasis on examples, Welby showed how practical issues affected the details of each case. As a result of their correspondence, Peirce revised and generalized the foundation of his logic, semeiotic, and pragmatism.

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Welby shared Peirce"s broad view of meaning and communication. In *What is Meaning* (1903), she wrote "There is, strictly speaking, no such thing as the Sense of a word, but only the sense in which it is used — the circumstances, state of mind, reference, 'universe of discourse' belonging to it". In the *Encyclopedia Britannica* (1911), she emphasized the "importance of acquiring a clear and orderly use of the terms of what we vaguely call Meaning; and also of the active modes, by gesture, signal or otherwise, of conveying intention, desire, impression and rational or emotional thought."

Whitehead and Wittgenstein would agree, but Frege, Russell, and their followers would strongly disagree. Among linguists, the founder of transformational grammar, Zellig Harris, wrote "We understand what other people say through empathy — imagining ourselves to be in the situation they were in, including imaging wanting to say what they wanted to say." But his star pupil, Noam Chomsky, would claim that empathy is outside the subject matter of linguistics.

5. Language Games

Peirce and Wittgenstein made a major transition from their early philosophy to their later, and both in the same direction. One critic said that Wittgenstein began as a logician and ended as a lexicographer. Ironically, that remark, which was intended in a derogatory sense, is true in a higher sense: they both discovered the flexibility and expressive power of natural languages. For Peirce, the transition was marked by the 16,000 definitions he wrote or edited for the *Century Dictionary*. For Wittgenstein, it was his second published book, *Wörterbuch für Kindern*, which he wrote when he was teaching elementary school in Austrian mountain villages. He learned that children do not think or speak along the lines of his first book, the *Tractatus Logico-Philosophicus* (TLP).

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6. Diagrams As the Language of Thought

Peirce's writings on logic, semeiotic, and diagrammatic reasoning, which had been neglected for most of the 20th century, are now at the forefront of research in the 21st. The psychologist Johnson-Laird (2002), who had written extensively about mental models, said that Peirce's existential graphs and rules of inference are a good candidate for a neural theory of reasoning:

> Peirce's existential graphs are remarkable. They establish the feasibility of a diagrammatic system of reasoning equivalent to the first-order predicate calculus. They anticipate the theory of mental models in many respects, including their iconic and symbolic components, their eschewal of variables, and their fundamental operations of insertion and deletion. Much is known about the psychology of reasoning... But we still lack a comprehensive account of how individuals represent multiply-quantified assertions, and so the graphs may provide a guide to the future development of psychological theories.

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These observations imply that cognition involves an open-ended variety of interacting processes. Frege's rejection of psychologism and "mental pictures" reinforced the behaviorism of the early 20th century. But the latest work in neuroscience uses "folk psychology" and introspection to interpret data from brain scans (Dehaene 2014). The neuroscientist Antonio Damasio (2010) summarized the issues:

> The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them.

The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires. They provide a "model theoretic" semantics for language that uses perception and action for testing models against reality. Like Tarski's models, they define the criteria for truth, but they are flexible, dynamic, and situated in the daily drama of life.

7. Diagrammatic Reasoning

Everybody thinks in diagrams — from children who draw diagrams of what they see to the most advanced scientists and engineers who draw what they think. Ancient peoples saw diagrams in the sky, and ancient monuments are based on those celestial diagrams. They correspond to the mathematical "patterns of plausible inference" identified by Pólya (1954). The role of diagrammatic reasoning is one of Peirce's most brilliant insights, and the generalized EGs in his late writings include much more than an alternative to predicate calculus.

> All necessary reasoning without exception is diagrammatic. That is, we construct an icon of our hypothetical state of things and proceed to observe it. This observation leads us to suspect that something is true, which we may or may not be able to formulate with precision, and we proceed to inquire whether it is true or not. For this purpose it is necessary to form a plan of investigation, and this is the most difficult part of the whole operation. We not only have to select the features of the diagram which it will be pertinent to pay attention to, but it is also of great importance to return again and again to certain features. (EP 2:212)

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Computer systems can communicate with people by traslating their internal represenations to and from notations that people can read and understand. But as Zelling Harris said, computers cannot understand what people say until they have sufficient empathy to imagine themselves to be in the situations the humans are in, including imaging wanting to say what the humans want to say.

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