An excerpt from a forthcoming article by John Sowa, 28 September 2023..

1. Developments from 1903 to 1913

For Peirce, 1902 brought an end to two major projects: Baldwin's dictionary was finished, and funding for his *Minute Logic* was rejected. But three events in 1903 led him to rethink every aspect of his life's work: his Harvard lectures in the spring, his Lowell lectures in the fall, and his correspondence with Victoria Welby. As a guide to the new developments, the tree in Figure 1 shows his classification of the sciences and dependencies among them. Branches show the classification, and dotted lines show the dependencies. Sciences to the right of each dotted line depend on sciences to the left. Pure mathematics stands alone, and all other sciences and engineering depend on mathematics (CP 1.180ff, 1903).

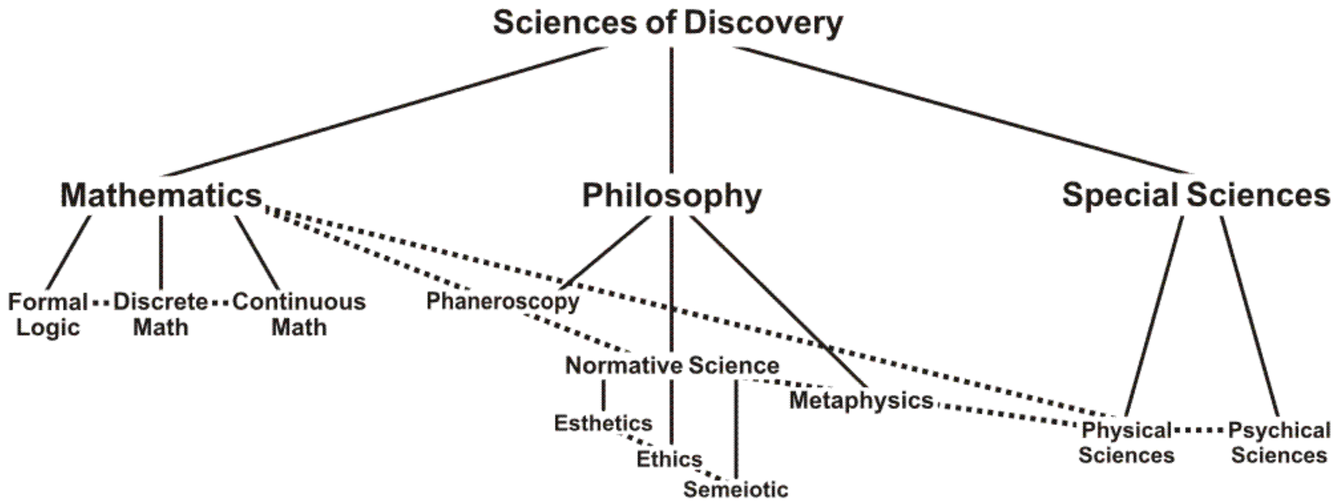

Figure 1: Classification of the Sciences and Dependencies Among Them

As Carl Gauss said, mathematics is the queen of the sciences. It includes an infinity of theories of all possible, imaginable, or observable patterns. For any physical pattern, some mathematical theories can approximate it at various levels of precision. The challenge for science is to discover them. To meet the challenge, Peirce proposed mathematical diagrams called *existential graphs* (EGs). They can map patterns of observation to patterns for thinking, reasoning, acting, or speaking.

Peirce's sequence of phaneroscopy, normative science, and metaphysics is consistent with Aristotle's *Metaphysics*. Book Alpha begins "All people (*pantes anthropoi*) by nature (*physei*) reach for (*oregontai*) knowledge (*tou eidenai*)." It continues with an analysis of what people and other animals experience. That corresponds to Peirce's definition of phaneroscopy: the analysis of "all that is in any way or in any sense present to the mind, quite regardless of whether it corresponds to any real thing or not" (CP 1.284). Reaching for knowledge implies desire (a value judgment) for determining what is true. That would require Peirce's methodeutic, which is a branch of logic as semeiotic. Aristotle also discussed the normative values of beauty (*kalos*) and the good (*agathon*) before analyzing the nature of being and the categories of entities (ontology).

As his ideas evolved, Peirce revised his terminology to express the fine points more precisely. In 1904, he replaced Kant's term *phenomenology* with *phaneroscopy.* For the three branches of normative science, he chose the names *ethics*, *esthetics*, and *logic as semeiotic* or just *logic*. In modern textbooks, however, the word *logic* means *formal logic*. To avoid confusion, Max Fisch (1981) chose *semeiotic* for that branch. This article follows that practice.

[The remainder of Section 1 is omitted.]

2. The Role of Diagrams in Phaneroscopy

For the third *Monist* article, *Prolegomena to an Apology for Pragmaticism*, Peirce chose a title that echoes Kant's *Prolegomena*. In it, he addressed Kant's three "transcendental questions": How is pure mathematics possible? How is pure natural science possible? How is metaphysics in general possible? The dotted lines in Figure 1 suggested the answer shown in Figure 2: diagrams, such as EGs, are mathematical structures that relate phaneroscopy, metaphysics, and the natural sciences to methods for thinking, talking, and acting in and on the world.

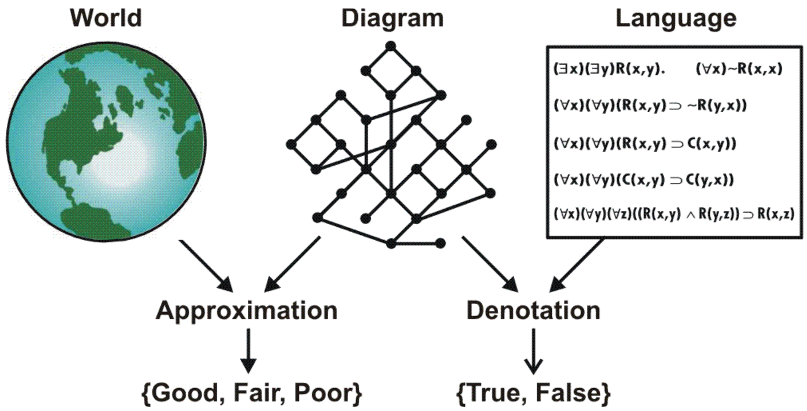

Figure 2: Diagrams relate thought and language to the world

The first sentence sets the stage: "Come on, my Reader, and let us construct a diagram to illustrate the general course of thought; I mean a System of diagrammatization by means of which any course of thought can be represented with exactitude" (CP 4:530). Figure 2 shows an important step beyond Tarski's model theory. Instead of a one-step mapping from the world to language, the diagram splits the mapping in two distinct steps.

Phaneroscopy maps some aspect of the world to a diagram, which is "an icon of a set of rationally related objects" (R293, NEM 4:316). It serves as a Tarski-style model for determining the denotation of languages, formal or informal. But when a continuous world is mapped to a discrete diagram, an enormous amount of detail is lost. Although the right side can be a precise map from a graph to a formal logic, it may be an approximate mapping from an informal diagram to the informal languages that people speak. In his career as a mathematician, logician, philosopher, physicist, chemist, biologist, linguist, lexicographer, and engineer, Peirce understood the complexity of both sides.

To deal with that complexity, Peirce "widened... the familiar logical triplet [of] Term, Proposition, Argument" to *seme*, *pheme*, and *delome*. A seme may be a logical term, or it may be an image or diagram that resembles something in the world. A pheme may be a proposition stated in some language, or it may be a pattern of semes that shows what some proposition states. A delome is a sequence of phemes that states an argument. It may be represented by "diagrams imagined to be phenakisticopically combined" (R292b). Today, virtual reality is a better term for that combination.

With this terminology, Peirce renamed the EG *sheet of assertion* as a *phemic sheet*, which "iconizes the Universe of Discourse [UoD], since it more immediately represents a field of Thought, or Mental Experience, which is itself directed to the Universe of Discourse, and considered as a sign, denotes that Universe" (R300). Figure 3 shows a phemic sheet derived by perception of the world and action on it.

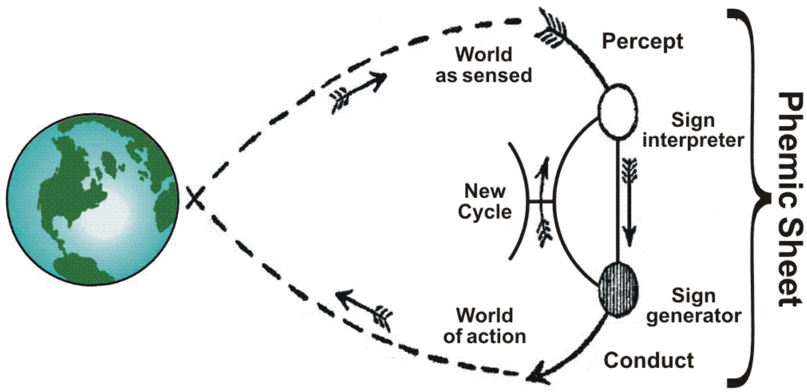

Figure 3: Deriving a phemic sheet by perception and action

Figure 3, adapted from a drawing by Uexküll (1920), shows how an animal of any species could sense and act upon the world. The "Mental Experience" (*Innenwelt*) of the animal is represented by a phemic sheet. The sign interpreter receives percepts (semes) from any external source or any organ in the body. A simple stimulus-response would take milliseconds to relate a sensory seme to a seme that triggers an action. But repeated cycles would relate and combine semes and phemes for diagrammatic reasoning. A delome would be a sequence of phemes that answers a question or resolves a dispute. In effect, Peirce and Uexküll anticipated the hypothesis of *artificial causation* by Craik (1943):

> If the organism carries a 'small-scale model' of external reality and of its own possible actions within its head, it is able to try out various alternatives, conclude which is the best of them, react to future situations before they arise, utilize the knowledge of past events in dealing with the present and future, and in every way to react in a much fuller, safer, and more competent manner to the emergencies which face it. (p. 61)

Peirce would approve of Uexküll's *biosemiotic* approach, since he wrote "The action of a sign generally takes place between two parties, the utterer and the interpreter. They need not be persons... many kinds of insect, and even plants make their livings by uttering signs" (R318, 1907). And Uexküll would agree with Peirce: "As for the senses of my dog, I must confess that they seem very unlike my own... I reflect to how small a degree he thinks of visual images, and of how smells play a part in his thoughts and imaginations analogous to the part played by sights in mine" (CP 1.314).

Before Peirce coined the words *seme, pheme,* and *delome,* he had introduced that trichotomy as "first, simple signs, substitutive signs, or Sumisigns; second, double signs, informational signs, quasi-propositions, or Dicisigns; third, triple signs, rationally persuasive signs, arguments, or Suadisigns" (R478, EP 2:275, 1903). He coined even more words to describe details of the sumisigns, which he more often called *rhemes*:

> Any material image, as a painting, is largely conventional in its mode of representation; but in itself, without legend or label, it may be called a *hypoicon*. Hypoicons may roughly [be] divided according to the mode of Firstness which they partake. Those which partake the simple qualities, or First Firstnesses, are *images*; those which represent the relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts, are *diagrams*; those which represent the representative character of a representamen by representing a parallelism in something else, are metaphors (EP 2:273-274).

Peirce also called the third kind of hypoicons *examples* "in respect to their intellectual characters." In his publications, however, he rarely analyzed enough examples to illustrate all the options for applying his terminology. But the correspondence with Welby coincided with his transition from an abstract phenomenology to a more concrete phaneroscopy. To relate his abstract theories to her writings, he moved away from an abstract Kantian style to illustrate his terminology with specific examples.

Recent studies of phaneroscopy have applied Peirce's methods to a wide range of examples and methods of reasoning. Jappy (2019) analyzed examples of hypoicons for representing the details of pictures, images, and gestures. Stjernfelt (2007, 2022) analyzed and compared Peirce's work to writings by phenomenologists from Husserl to the present on topics ranging from commonsense reasoning to advanced mathematics and psychological issues in anthropology and biosemiotics.

After analyzing the role of existential graphs in phaneroscopy, Bellucci (2015) concluded "What logicians call 'logical analysis' is, for Peirce, phaneroscopical analysis applied to logic." That point reflects the two-sided nature of existential graphs. As diagrams, they simplify the mapping from observations. As logic, they support all forms of induction, abduction, and deduction. For Peirce, the trichotomy of seme, pheme, and delome is the starting point of his *Prolegomena*. His answer to Kant's question "How is pure natural science possible?" is *methodeutic*, as Fisch observed (1986, p. 375):

> Peirce says "But pragmatism is plainly, in the main, a part of methodeutic" (R320:24) and "Pragmatism is, thus ... a mere rule of methodeutic, or the doctrine of logical method" (R322:13). Of course methodeutic depends on speculative grammar and on critic, and the way to pragmatism will have been cleared in these first two parts of semeiotic. That is, they will have made their contributions to the "proof."

A logic of the future should enable a proof of pragmatism. Peirce stated the requirements in his *Prolegomena*: "a System of diagrammatization by means of which any course of thought can be represented with exactitude." Then "operations upon diagrams, whether external or imaginary, take the place of the experiments upon real things that one performs in chemical and physical research." The system requires four components: (1) diagrams in EGs or other notations; (2) grammars for mapping languages to and from diagrams; (3) logical methods for reasoning about the diagrams; and (4) methods of perception and action for relating the world to diagrams and evaluating the results.

[The remainder of this section is omitted.]

3. Relating Images to Diagrams

Since the semes and phemes that flow along the arrows of Figure 4 may contain uninterpreted percepts and images, ordinary existential graphs cannot represent them. In the letter L231, in which Peirce specified his most general notation for EGs, he mentioned his hopes of representing "stereoscopic moving images." To accommodate them, Sowa (2016, 2018) proposed *generalized* existential graphs (GEGs). Figure 5 shows Euclid's Proposition 1 stated in three kinds of GEGs: "On a given finite straight line, to draw an equilateral triangle."

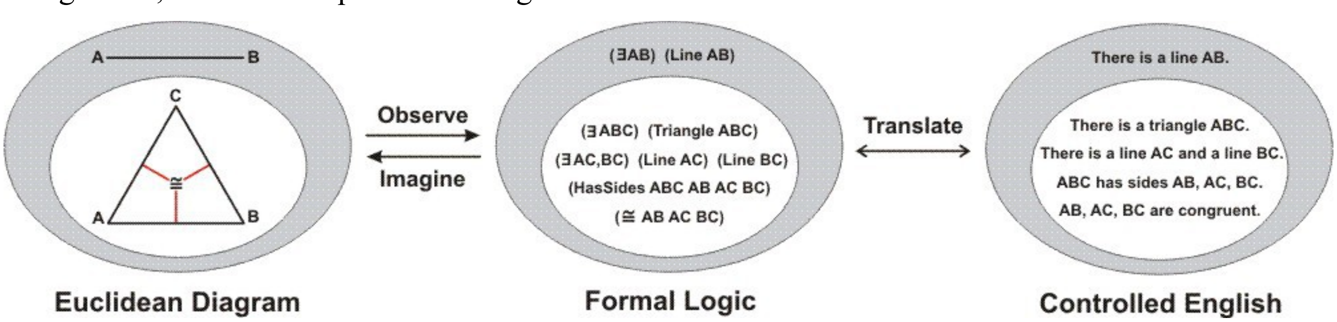

Figure 5: Euclid's Proposition 1 stated in three kinds of generalized EGs

In GEGs, shading represents negation. An area with no negations or an even number of negations is unshaded. In Figure 5, each GEG has a nest of two negations, not(*p* and not *q*). This pattern is equivalent to an implication: If *p* then *q*. In the GEG on the left, *p* and *q* are Euclid's diagrams for Proposition 1: If there is a line AB, then there is an equilateral triangle ABC. In the GEG in the middle, *p* and *q* are written in a linearized notation for EGs. In the GEG on the right, *p* and *q* are written in a version of controlled English that has an exact translation to the logic in the middle. Each of the three arrows represents a rule of inference that maps one kind of GEG to another:

- Observe: Convert an iconic GEG (image or diagram) to a GEG that may lose information. This rule corresponds to one or two standard EG rules: an iteration that produces an equivalent GEG (iconic or symbolic) followed by an optional erasure that loses information.
- Imagine: Convert a GEG to an iconic GEG that may gain information. This rule corresponds to one or two standard EG rules: an iteration that produces an equivalent iconic GEG followed by an optional insertion that gains information.
- Translate: Convert a symbolic GEG (diagram or language) to another symbolic GEG that may gain or lose information. This rule corresponds to one, two, or three standard EG rules: an iteration that produces an equivalent GEG followed by an optional insertion that gains information and an optional erasure that loses information.

The three arrows in Figure 5 do not perform any optional insertions or deletions. Therefore, they perform equivalent conversions from one kind of GEG to another. Equivalent conversions may be performed in any area, positive or negative. Conversions that gain information may be performed in any negative area. Conversions that lose information may be performed in any positive area. Conversions that gain and lose information are vague.

Figure 6 shows three kinds of semes: an image, an existential graph, and a phrase in controlled French. By the rule of observation, the image is converted to an EG that has lost many details. The translation from the EG to controlled French is an equivalence. A person who understands French could do the reverse translation to an equivalent EG. The reverse of observation is imagination, but no one is likely to imagine the exact details that were lost by the observation rule.

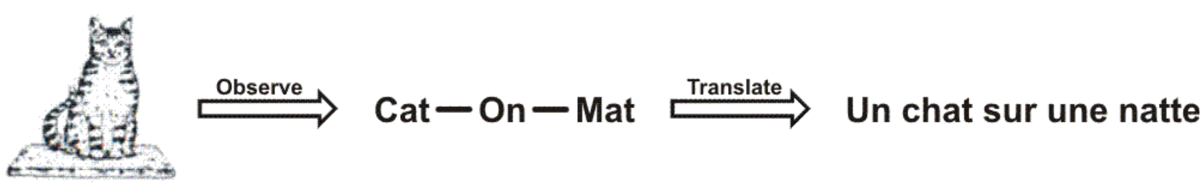

Figure 6: Converting semes by observation and translation

The GEG rules are based on Peirce's writings about mapping EGs to and from other forms. He explained that an iconic sign, such as an image or a diagram, is necessary for understanding symbols. Stereoscopic moving images would require the shaded and unshaded areas to be extended to regions in more than two dimensions. Representing synechism (continuity) and tychism (probability) would require more complex mathematics than an ordinary graph:

> The purpose of a Diagram is to represent certain relations in such a form that it can be transformed into another form representing other relations involved in those first represented. [Then] this transformed icon can be interpreted in a symbolic statement. (LNB 286r, 1906).

[The remainder of this section and all of sections 4 and 5 have been omitted.]

6. Searching for a Language of Thought

Although language is important for human communication, the systems of perception and action evolved millions of years before some early hominin began to talk. Monkeys and apes use vocalization primarily for expressing emotions. But the great apes use complex gestures for communication. All four species have learned subsets of human sign language, and they can understand a larger subset of human spoken language than they express with their signs. What is the cognitive foundation for their sign language? Could it be similar to the foundation for human language? Could a moving three-dimensional sign language support diagrammatic reasoning? Could it support Peirce's goal of "thinking in stereoscopic moving pictures?" (NEM 3:191)

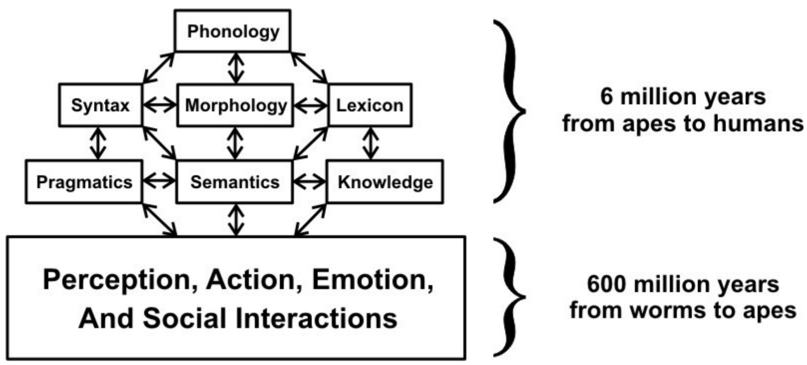

Figure 12: Did human language evolve from the signed languages of the apes?

Spoken languages, which replaced the signed languages of the apes, are usually accompanied by gestures and pointing. When added to speech, they increase its precision and expressive power. The signed languages of the deaf are diagrams in motion. They have the expressive power of speech with gestures and pointing. After years of research, the psycholinguist David McNeill (1992) recognized the role of gestures: relate the content of the discourse to the context and the speaker's intentions.

> Images and speech are equal and simultaneously present in the mind... Gestures look upward, into the discourse structure, as well as downward, into the thought structure. A gesture will occur only if one's current thought contrasts with the background discourse. If there is a contrast, how the thought is related to the discourse determines what kind of gesture it will be, how large it will be, how internally complex it will be, and so forth.

As a rich system of gestures, the signed languages of the deaf take advantage of multidimensional moving signs to support expressive power comparable to spoken languages. The index finger is an indexical that replaces pronouns by pointing. For references to people and things that left the scene, the signer points to where they had been. The signer can also introduce new characters and things, place them in fixed locations in the air, and refer to them by pointing. With moving gestures the signer can show motion and direction.

The same brain areas that support spoken languages also support signed languages, but areas that support vision and motion are also involved (Campbell et al. 2007). Hearing adults who also learn a signed language become bimodal. Emmorey and MacSweeney (2009) discovered that their brain activation patterns are intermediate between those of deaf signers and hearing non-signers. On tests of mental imagery (generating, rotating, remembering, and matching 3-D shapes), they score higher than non-signers. Those skills, which are important for apes swinging through trees, are the kind that Peirce wanted to represent. In effect, gestures are moving diagrams that can relate imagery and actions to languages of any kind.

In a study of bilingual infants whose parents speak or sign different languages, Petitto (2005) discovered significant similarities and differences. She studied subjects whose parents spoke or signed all six combinations of four languages: English, French, American Sign Language (ASL), and Langue des Signes Québécoise (LSQ). Monolingual and bilingual babies go through the same stages and at the same ages for both spoken and signed languages. Hearing babies born to profoundly deaf parents babble with their hands, but not vocally. Babies bilingual in a spoken and a signed language babble in both modalities, vocally and with their hands. And they express themselves with equal fluency in their spoken and signed language at every stage of development.

To express the semantics of both kinds of language, moving 3-D diagrams are more versatile than a static linear form. The cerebral cortex consists of interacting areas for perception, action, learning, reasoning, emoting, and communicating. As Barsalou (2008) wrote, cognition is "embedded in, distributed across, and inseparable from" those processes. When people view a static object, they anticipate working with it. When people view food, they anticipate its taste when eating it. Musicians identify their own performances by recognizing the fingering. Visual and motor simulations are essential to language understanding. Affect, feelings, rewards, and value judgments are fundamental to all aspects of reasoning and decision making. No single aspect is cognition, but all of them together are cognition. Social interactions facilitate learning by stimulating more aspects.

These observations are consistent with the views of a pioneer in artificial intelligence, Minsky (1965): no single mechanism, by itself, can adequately support the full range of functions required for a human level of intelligence. He coined the term *Society of Mind* (1986) to characterize that insight:

> What magical trick makes us intelligent? The trick is that there is no trick. The power of intelligence stems from our vast diversity, not from any single, perfect principle. Our species has evolved many effective although imperfect methods, and each of us individually develops more on our own. Eventually, very few of our actions and decisions come to depend on any single mechanism. Instead, they emerge from conflicts and negotiations among societies of processes that constantly challenge one another.

Neuroscientists have identified areas of the brain specialized for different aspects of perception, thought, and action. Repeated daily exercises develop smaller more specialized areas. Albert Einstein, for example, had an enlarged area of his brain for the fingers of his left hand because he played the violin daily. A squirrel's hippocampus grows larger in the fall when it must remember where it buried nuts for the winter; that area shrinks in the spring as it retrieves its cache. When London taxi drivers had to memorize the names and locations of every street, they also had an enlarged hippocampus. But when they got the information from satellites, their hippocampus did not grow larger.

Figure 13 shows the lobes of the left hemisphere of the brain and the aspects of cognition they control (MacNeilage 2008). Peirce's phaneron, the first stage of cognition, is processed in the primary visual cortex at the back of the occipital lobes. But those lobes also process much more than vision. Even people who have been blind since birth can generate moving three-dimensional mental imagery by combining information from other sensory modalities. The occipital place area (OPA), just above the primary visual cortex, seems to be the location for the combinations. For transmision to other areas of the brain, mental images may be simplified to mental diagrams, which could be represented by gestures in sign languages or by generalized EGs in a computer.

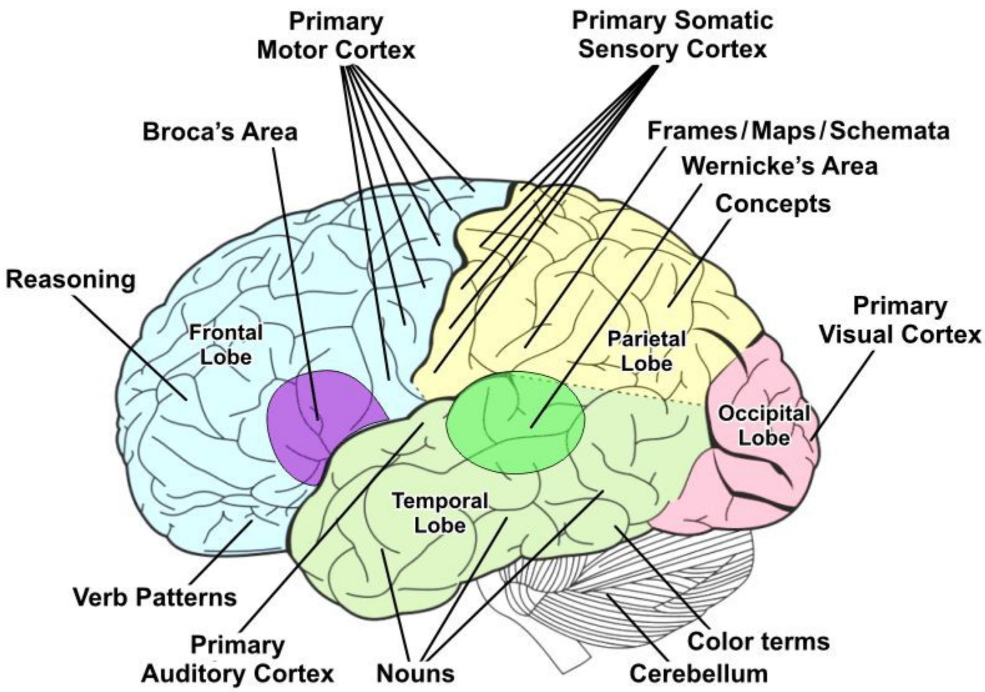

Figure 13: Brain regions of the left hemisphere

Temporal lobes, located behind the ears, process sounds and relate them to imagery in the occipital lobes. They also recognize certain sounds as words, which they relate to Wernicke's area, which is responsible for language interpretation and generation. The parietal lobes generate and process patterns of connections among concepts derived from any source: images from the occipital lobes, sounds and words from the temporal lobes, and feelings from the sensory cortex, which is connected to all parts of the body. These patterns, which correspond to Peirce's diagrams, may also be called maps, frames, schemata, or semantic networks.

The frontal lobes are involved in all intentional or purposive thought and action. The primary motor cortex for controlling action is in the frontal lobes, parallel to the sensory cortex in the parietal lobes. The area for verbs is in the left frontal lobe, but the areas for nouns are in the temporal lobe. Broca's area for generating language, spoken and signed, is connected to Wernicke's area by a thick bundle of nerve fibers called the *arcuate fasciculus*. Both of its ends branch out to areas in the front and rear of the cortex. The prefrontal cortex, which is especially large in humans, is responsible for all complex thought and reasoning. The disastrous operation called a *prefrontal lobotomy*, which was intended to cure mental disorders, left patients with a listless, meaningless life.

The large cerebral cortex has about 16 billion neurons. The much smaller cerebellum, as shown in Figure 13, takes only 10% of the volume of the brain, but it has about 70 billion tightly packed neurons (Herculano-Houzel 2012). For years, scientists thought that its primary role was to control movement, but it also plays a major role in perception, cognition, language, planning, emotion, social interactions, and even mathematical reasoning. The computational power and precision that enables primates to leap through trees also supports the most complex calculations by sedentary mathematicians. Nothing in the cerebellum is conscious, but its computations affect all conscious experience.

A remarkable experiment produced three kinds of fMRI scans (Figure 14). They show monadic patterns in perception (scan #1), dyadic patterns in thinking about structure (#2), and triadic patterns in thinking about causality (#3). To avoid showing the language activity in the left hemisphere, Figure 14 shows only the thinking patterns in the right hemisphere. For perception, fMRI scan #1 shows no noticeable activation outside the visual cortex. For structure, scan #2 shows two bright patterns that relate a large area of the occipital lobe to the the sensory cortex in the parietal lobe. For causality, scan #3 shows that the visual, parietal, and frontal lobes are active; the brightest area includes the sensory and action areas of the parietal and frontal lobes. Areas toward the front are also active for reasoning about the interconnections. Those areas have connections to the cerebellum (lower left in Figure 14).

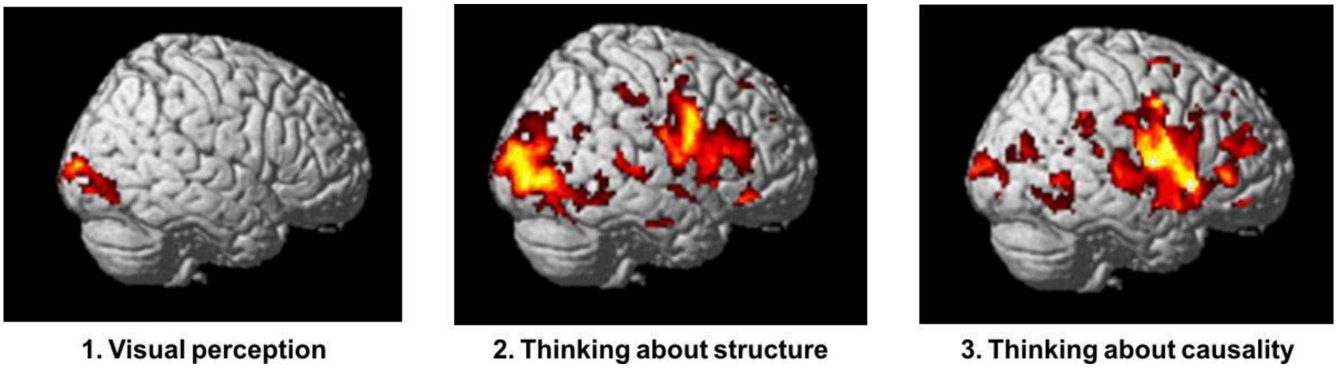

Figure 14: Three fMRI scans of the right hemisphere (Mason & Just, 2015)

For the experiment that produced those scans, Mason and Just selected subjects who were not familiar with mechanical devices: college students that did not major in STEM subjects (science, technology, engineering, or math). All of them studied four devices and their inner mechanisms: bathroom scale, fire extinguisher, disc brake system, and trumpet. During test sessions, an fMRI scanner recorded patterns of brain activity. An early training session just showed pictures and named the parts: "A bathroom scale consists of a spring, a lever, a ratchet, and a dial." Later sessions explained structural and causal relations: "The spring pulls a ratchet which rotates a gear attached to a measurement dial."

The three fMRI scans in Figure 14 show experimental evidence for Peirce's *phenomenological categories* of Firstness, Secondness, and Thirdness. The first scan of visual perception shows monadic patterns in the visual cortex. They are the unprocessed or slightly processed images that Peirce called the *phaneron*. The second scan of thinking about structure shows dyadic patterns that relate activations in the parietal lobes and the occipital lobes. The brightest area is the occipital place area (OPA), which combines information from internal and external sources. That may be the area where mental models or Peirce's diagrams are constructed. The third scan of thinking about causality shows triadic patterns that relate causal intentions in the frontal lobes to activations in the parietal and occipital lobes.

These observations show that cognition involves a wide variety of interacting processes in all parts of the brain. Frege's rejection of psychologism and "mental pictures" reinforced the behaviorism of the early 20th century. But recent work in neuroscience uses "folk psychology" and introspection to interpret data from brain scans. In summarizing the issues, the neuroscientist Antonio Damasio (2010) emphasized that imagery is fundamental:

> The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them.

The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires. They provide a "model theoretic" semantics for thought that uses perception and action for testing models against reality. Like Tarski's models, maps and images determine the criteria for truth, but they are flexible, dynamic, and situated in the daily drama of life. To represent all aspects of thought, they require the features summarized in Section 2 and defined in Section 3.

7. Diagrammatic Reasoning

Everybody thinks in diagrams — from children who draw diagrams of what they see to the most advanced scientists and engineers who draw what they think. Ancient peoples saw diagrams in the sky, and ancient monuments are based on those celestial diagrams. They correspond to the mathematical "patterns of plausible inference" identified by Pólya (1954). The role of diagrammatic reasoning is one of Peirce's most brilliant insights, and the patterns of diagrams support every kind of reasoning from vague analogies to the most precise deductions:

> We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular... This diagram, which has been constructed to represent intuitively or semi-intuitively the same relations which are abstractly expressed in the premisses, is then observed, and a hypothesis suggests itself that there is a certain relation between some of its parts — or perhaps this hypothesis had already been suggested. In order to test this, various experiments are made upon the diagram, which is changed in various ways. (CP 2.778)

Peirce's writings on logic, semeiotic, and diagrammatic reasoning, which had been neglected for most of the 20th century, are now at the forefront of research in the 21st. The psychologist Johnson-Laird (2002), who had written extensively about mental models, said that Peirce's existential graphs and rules of inference are a good candidate for a neural theory of reasoning:

> Peirce's existential graphs are remarkable. They establish the feasibility of a diagrammatic system of reasoning equivalent to the first-order predicate calculus. They anticipate the theory of mental models in many respects, including their iconic and symbolic components, their eschewal of variables, and their fundamental operations of insertion and deletion. Much is known about the psychology of reasoning... But we still lack a comprehensive account of how individuals represent multiply-quantified assertions, and so the graphs may provide a guide to the future development of psychological theories.

For board games like chess, diagrammatic reasoning is the essence of the game. Most chess experts can play a good blindfold game. For them, the board and pieces represent a diagram in Peirce's sense, and their strategies are an example of Pólya's patterns of plausible inference. In describing his way of thinking, Einstein used Peirce's words *visual* and *muscular*:

> The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be voluntarily reproduced and combined... The above-mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be reproduced at will. (Quoted by Hadamard, 1945)

Over the years, Peirce added further observations about the methods of diagrammatic reasoning:

> All necessary reasoning without exception is diagrammatic. That is, we construct an icon of our hypothetical state of things and proceed to observe it. This observation leads us to suspect that something is true, which we may or may not be able to formulate with precision, and we proceed to inquire whether it is true or not. For this purpose it is necessary to form a plan of investigation, and this is the most difficult part of the whole operation. We not only have to select the features of the diagram which it will be pertinent to pay attention to, but it is also of great importance to return again and again to certain

> features. (EP 2:212)
>
> The word *diagram* is here used in the peculiar sense of a concrete, but possibly changing, mental image of such a thing as it represents. A drawing or model may be employed to aid the imagination; but the essential thing to be performed is the act of imagining. Mathematical diagrams are of two kinds; 1st, the geometrical, which are composed of lines (for even the image of a body having a curved surface without edges, what is mainly seen by the mind's eye as it is turned about, is its generating lines, such as its varying outline); and 2nd, the algebraical, which are arrays of letters and other characters whose interrelations are represented partly by their arrangement and partly by repetitions. If these change, it is by instantaneous metamorphosis. (NEM 4:219)
>
> Diagrammatic reasoning is the only really fertile reasoning. If logicians would only embrace this method, we should no longer see attempts to base their science on the fragile foundations of metaphysics or a psychology not based on logical theory. (CP 4.571)

With that last quotation, Peirce dismissed Frege's criticism of psychologism. Logic is not based on psychology, but psychology is based on logic. The methods of diagrammatic reasoning apply to every branch of science in Figure 1: from mathematics and phaneroscopy to every branch of philosophy, physical sciences, psychical sciences, practical reasoning, and common sense.

Note: This section will conclude with a presentation of diagrammatic reasoning for induction, abduction, deduction, and analogy. It will also analyze the Large Language Models (LLMs), which have been very useful for machine translation and for natural language interfaces to computer systems. LLMs can also be useful for suggesting hypotheses that must be verified by formal methods, such as diagrammatic reasoning. But they cannot do the full range of diagrammatic reasoning.

For an introduction to Peirce's existetenial graphs and some of the issues in Sections 1 to 5 of this article, see slides 13 to 34 of *Language, ontology, and the Semantic Web*, a lecture at the 2020 European Semantic Web Conference, https://jfsowa.com/talks/eswc.pdf

For more detail about material that is summarized in Section 3 of this article, see the article, Reasoning with diagrams and images, *Journal of Applied Logics* **5:5**, 987-1059, by John Sowa (2018). http://www.collegepublications.co.uk/downloads/ifcolog00025.pdf