Excerpts from Sections 2, 3, and 4 of an article by John Sowa, 17 November 2023.

Diagrams serve as a bridge between images and languages, formal or informal. Figure 2 shows an important step beyond Tarski's model theory. Instead of a one-step mapping from the world to a language, the diagram splits the mapping in two distinct steps.

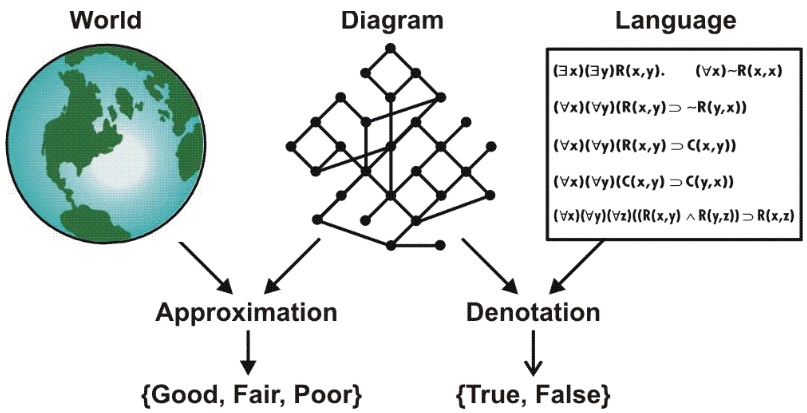

Figure 2: Diagrams relate the world to languages that describe it

The first step, which Peirce called *phaneroscopy* maps images of the world to diagrams, each of which is "an icon of a set of rationally related objects" (NEM 4:316). They serve as Tarski-style models for determining the denotation of languages, formal or informal. But when a continuous world is mapped to a discrete diagram, an enormous amount of detail is lost.

The right side may be a precise map from a formal diagram to a formal logic. More often, it maps an informal diagram to the informal languages that people speak. In his career as a mathematician, logician, philosopher, physicist, chemist, biologist, linguist, lexicographer, and engineer, Peirce understood the complexity of both sides. To deal with the complexity, he "widened... the familiar logical triplet of Term, Proposition, Argument" to *seme*, *pheme*, and *delome*.

A seme may be a logical term, or it may be an image or diagram that resembles something in the world. A pheme may be a proposition stated in some language, or it may be a pattern of semes that shows what some proposition states. A delome is a sequence of phemes that explains something. As a representation,, Peirce defined a *phemic sheet*, which "iconizes the Universe of Discourse" (R300). Figure 3 shows a phemic sheet derived by perception of the world and action on it.

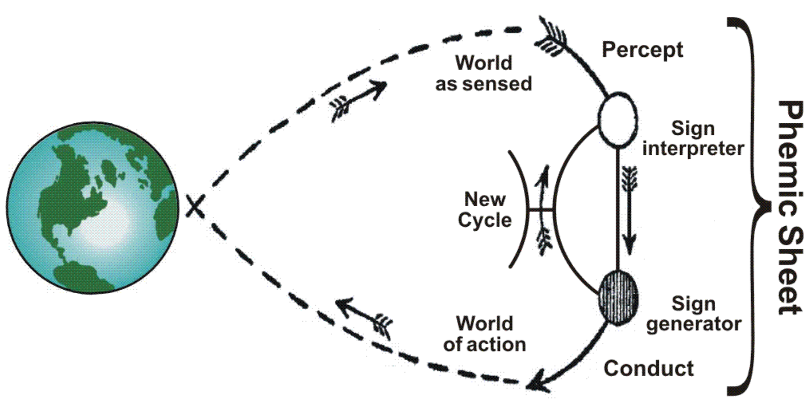

Figure 3: Deriving a phemic sheet by perception and action

Figure 3, adapted from a drawing by Uexküll (1920), shows how an animal of any species could sense and act upon the world. The "Mental Experience" (*Innenwelt*) of the animal is represented by a phemic sheet. The sign interpreter receives percepts (semes) from any external source or any organ in the body. A simple stimulus-response would take milliseconds to relate a sensory seme to a seme that triggers an action. But repeated cycles would relate and combine semes and phemes for diagrammatic reasoning. A delome would be a sequence of phemes that answers a question or resolves a dispute. In effect, Peirce and Uexküll anticipated the hypothesis of *artificial causation* by Craik (1943):

> If the organism carries a 'small-scale model' of external reality and of its own possible actions within its head, it is able to try out various alternatives, conclude which is the best of them, react to future situations before they arise, utilize the knowledge of past events in dealing with the present and future, and in every way to react in a much fuller, safer, and more competent manner to the emergencies which face it. (p. 61)

Peirce would approve of Uexküll's *biosemiotic* approach, since he wrote "The action of a sign generally takes place between two parties, the utterer and the interpreter. They need not be persons... many kinds of insect, and even plants make their livings by uttering signs" (R318, 1907). And Uexküll would agree with Peirce: "As for the senses of my dog, I must confess that they seem very unlike my own... I reflect to how small a degree he thinks of visual images, and of how smells play a part in his thoughts and imaginations analogous to the part played by sights in mine" (CP 1.314).

After analyzing the role of existential graphs in phaneroscopy, Bellucci (2015) concluded "What logicians call 'logical analysis' is, for Peirce, phaneroscopical analysis applied to logic." The analysis involves two steps in mapping the phaneron to existential graphs (EGs). The first maps aspects of mental experience to images, diagrams, metaphors, and examples, which Peirce called *hypoicons*. The second maps hypoicons to the semes, phemes, and delomes represented in EGs.

> Any material image, as a painting, is largely conventional in its mode of representation; but in itself, without legend or label, it may be called a *hypoicon*. Hypoicons may roughly [be] divided according to the mode of Firstness which they partake. Those which partake the simple qualities, or First Firstnesses, are *images*; those which represent the relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts, are *diagrams*; those which represent the representative character of a representamen by representing a parallelism in something else, are *metaphors* (EP 2:273).

In his studies of hypoicons, Jappy (2019) developed methods for representing the details of pictures, images, and gestures. Stjernfelt (2007, 2022) compared the full range of Peirce's writings on signs and diagrams to writings by phenomenologists from Husserl to the present He showed that Peirce's writings are at the forefront of current research in biosemiotics and cognitive science.

As diagrams, EGs support the mappings illustrated in Figure 2. The trichotomy of seme, pheme, and delome is the starting point of Peirce' *Prolegomena*. His answer to Kant's question "How is pure natural science possible?" is *methodeutic*, as Fisch observed (1986, p. 375):

> Peirce says "But pragmatism is plainly, in the main, a part of methodeutic" (R320:24) and "Pragmatism is, thus ... a mere rule of methodeutic, or the doctrine of logical method" (R322:13). Of course methodeutic depends on speculative grammar and on critic, and the way to pragmatism will have been cleared in these first two parts of semeiotic. That is, they will have made their contributions to the "proof."

Since phaneroscopy maps the world to diagrams, Peirce compared diagrammatic reasoning to experiments on the world: "operations upon diagrams, whether external or imaginary, take the place of the experiments upon real things that one performs in chemical and physical research" (CP 4.530).

[Remainder of Section 2 omitted]

3. Relating Images to Diagrams

Peirce did not limit diagrams to existential graphs. He said that a portrait or drawing, by itself, could be a seme that represents a possibility. With names or labels, a drawing could be a pheme that asserts a proposition. In 1911, he talked about representing "stereoscopic moving images" (L231), and he later proposed extended EGs called *Delta graphs*. As a candidate for Delta graphs, *generalized* existential graphs (GEGs) may include hypericons (Sowa 2018). Figure 5 shows Euclid's Proposition 1 stated in three kinds of GEGs: "On a given finite straight line, to draw an equilateral triangle."

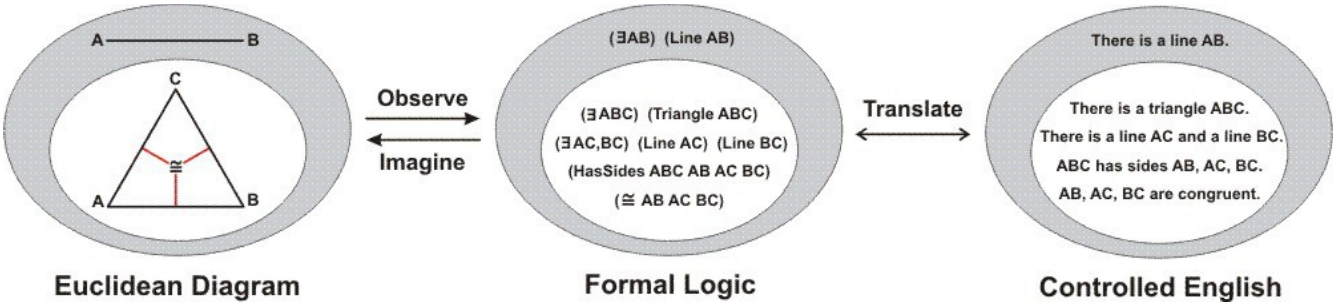

Figure 5: Euclid's Proposition 1 stated in three kinds of generalized EGs

In GEGs, shading represents negation. An area with no negations or an even number of negations is unshaded. In Figure 5, each GEG has a nest of two negations, not(p and not q). This pattern is equivalent to an implication: If p then q. In the GEG on the left, p and q are Euclid's diagrams for Proposition 1: If there is a line AB, then there is an equilateral triangle ABC. In the GEG in the middle, p and q are written in a linearized notation for EGs. In the GEG on the right, p and q are written in a version of *controlled* English that has an exact translation to the logic in the middle. Each of the three arrows represents a rule of inference that maps one kind of GEG to another:

- Observe: Convert an iconic GEG (image or diagram) to a GEG that may lose information. This rule corresponds to one or two standard EG rules: an iteration that produces an equivalent GEG (iconic or symbolic) followed by an optional erasure that loses information.
- Imagine: Convert a GEG to an iconic GEG that may gain information. This rule corresponds to one or two standard EG rules: an iteration that produces an equivalent iconic GEG followed by an optional insertion that gains information.
- Translate: Convert a symbolic GEG (diagram or language) to another symbolic GEG that may gain or lose information. This rule corresponds to one, two, or three standard EG rules: an iteration that produces an equivalent GEG followed by an optional insertion that gains information and an optional erasure that loses information.

The three arrows in Figure 5 do not perform any optional insertions or deletions. Therefore, they perform equivalent conversions from one kind of GEG to another. Equivalent conversions may be performed in any area, positive or negative. Conversions that gain information may be performed in any negative area. Conversions that lose information may be performed in any positive area. Conversions that gain and lose information are vague.

Figure 6 shows three kinds of semes: an image, an existential graph, and a phrase in controlled French. By the rule of observation, the image is converted to an EG that has lost many details. The translation from the EG to controlled French is an equivalence. A person who understands French could do the reverse translation to an equivalent EG. The reverse of observation is imagination, but no one is likely to imagine the exact details that were lost by the observation rule.

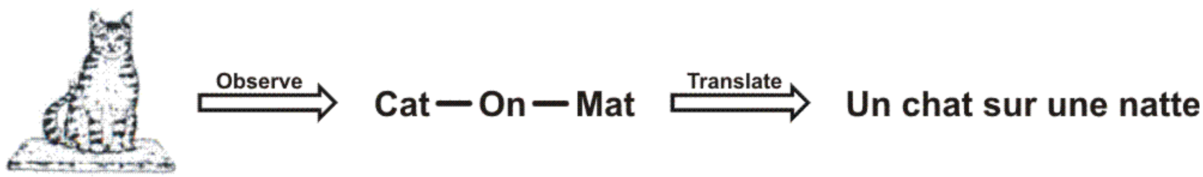

Figure 6: Converting semes by observation and translation

The GEG rules are based on Peirce's writings about mapping EGs to and from other forms. He explained that an iconic sign, such as an image or a diagram, is necessary for understanding symbols. Stereoscopic moving images would require the shaded and unshaded areas to be extended to regions in more than two dimensions. Representing synechism (continuity) and tychism (probability) would require more complex mathematics than an ordinary graph:

> The purpose of a Diagram is to represent certain relations in such a form that it can be transformed into another form representing other relations involved in those first represented. [Then] this transformed icon can be interpreted in a symbolic statement.
>
> It is necessary that the Diagram should be an Icon in which the inferred relation should be preserved. And it is necessary that it should be insofar General that one sees that accompaniments are no part of the Object. The Diagram is an Interpretant of a Symbol in which the signification of the Symbol becomes a part of the object of the icon. No other kind of sign can make a Truth *evident*. For the *evident* is that which is presented in an image, leaving for the work of the understanding merely the Interpretation of the Image in a Symbol (LNB 286r, 1906).

Besides EGs, Peirce had designed and used many kinds of diagrams throughout his career. Figure 7 shows how different kinds may be more or less iconic for different purposes. On the left is a musical diagram. On the right is a *conceptual graph* (CG) that represents equivalent information. For music, the traditional notation is more iconic because it highlights the melody. The CG buries the melody in a mass of distracting detail.

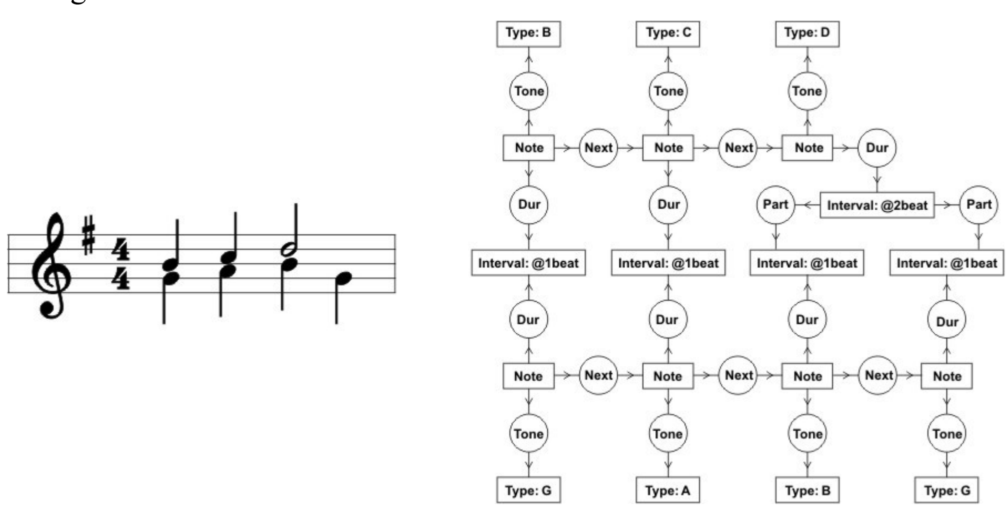

Figure 7. Two diagrams that represent the same musical passage

Conceptual graphs were designed as a formal logic for representing the semantics of natural languages, and every CG can be translated to and from an equivalent EG (Sowa 2008). Figure 8 shows five nodes from the upper left corner of the CG in Figure 7 and its translation to seven nodes of an equivalent EG. Both graphs may be translated to and from an equivalent sentence in controlled English: "Some note has a tone of type B and a duration of an interval 1 beat."

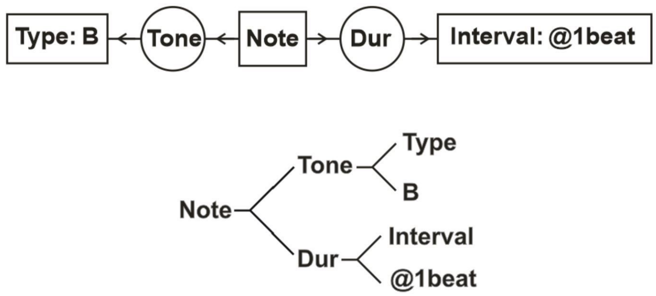

Figure 8. A conceptual graph and an equivalent existential graph

A CG is a *bipartite graph* with boxes that represent concepts and ovals that represent relations among the concepts. By default, a concept box has an existential quantifier ∃. For the CG in Figure 8, the quantifiers for the three concepts correspond to the three ligatures of the EG. This example illustrates the power of a well designed diagram. Just a single note in Figure 7 expands to an English sentence, the CG or the EG of Figure 8, or the following formula in predicate calculus:

$$(\exists x)(\exists y)(\exists z)(\mathrm{Note}(x) \wedge \mathrm{Tone}(x,y) \wedge \mathrm{Type}(y) \wedge y{=}B \wedge \mathrm{Duration}(x,z) \wedge \mathrm{Interval}(z) \wedge z{=}@1\mathrm{beat}.$$

These notations were designed for different purposes. The bar of music in Figure 7 is the most iconic for a musician to play. But it is too complex for efficient computer processing. Yet none of them represent the full rich sound that the composer intended and an expert musician can produce. The composer might add vague comments, such as *presto* (quickly) or *lento ma non troppo* (slowly but not too much). A recording (sound image) may be stored in some area of a GEG, but discrete symbols cannot precisely represent continuous sound.

With their rigid, unyielding precision, formal notations are never vague, but what they say so precisely, might not be correct or even relevant to the subject. Natural languages can approximate every step from a vague suggestion to a final resolution. In his work as an experimental physicist, a practicing engineer, and a lexicographer, Peirce understood the challenge:

> It is easy to speak with precision upon a general theme. Only, one must commonly surrender all ambition to be certain. It is equally easy to be certain. One has only to be sufficiently vague. It is not so difficult to be pretty precise and fairly certain at once about a very narrow subject. (CP 4.237. 1902)

This quotation summarizes the task of methodeutic: derive a precise theory from a series of approximate measurements. When scientists and engineers relate experiments to theories, they state estimated error bounds. But common sense tolerates vagueness. Accommodating both is the crux of pragmaticism.

To show how Peirce's rules may be adapted to different kinds of GEGs, Sowa (2018) compared four different proofs of Euclid's Proposition 1. The first is Heath's English translation of Euclid's Greek. The second is a step-by-step translation of Heath's English to controlled English, as in the right side of Figure 5. The third is a proof with Euclidean diagrams nested in EG areas, as in the left. The fourth is a proof in the symbolic notation in the middle. All three GEG proofs apply the same rules in the same sequence to equivalent notations.

The direct translation of Euclid's proofs to EG or GEG proofs shows that Peirce's rules have a better claim to the term *natural deduction* than Gentzen's. Over the years, many authors have used modern logics and rules of inference to prove Euclid's theorems. But none of those proofs can be mapped line-by-line to or from Euclid's proofs. Furthermore, an unsolved research problem about relating proofs in Gentzen's two systems was solved by a simple proof in terms of Peirce's rules (Sowa 2018).

The methodeutic of science requires metalevel reasoning to evaluate the observations of phaneroscopy. In 1898, Peirce introduced the notation of Figure 9 for reasoning about existential graphs (RLT p. 151). The EG inside the oval asserts the proposition "You are a good girl." The line of identity asserts the existence of a proposition *p* in some universe of discourse U. The word *that* relates *p* to U: "That you are a good girl is much to be wished."

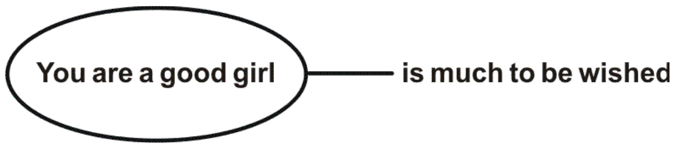

Figure 9. Peirce's notation for a metalevel statement

Wishing is one kind of possibility. Others include believing, hoping, fearing, wanting, planning, designing, intending, and avoiding. GEGs inside an oval can state information that is true in some universe of discourse. GEGs on the outside state metalevel information about the UoD. In English, the oval that marks an EG metalevel may be indicated by a conjunction *that* or an infinitive *to*.

Sentences that contain two or more verbs about beliefs, desires, and intentions may relate two or more UoDs. For example, the sentence *Tom believes that Mary wants to marry a sailor*, contains three clauses, whose nesting may be marked by brackets:

Tom believes [that Mary wants [to marry a sailor]].

The outer clause asserts that Tom has a belief, which is the proposition that Mary wants a situation, which is described by the infinitive *to marry a sailor*. Each clause makes a comment about the clause nested in it. References to the individuals mentioned in those clauses may cross context boundaries in various ways. The sailor, for example, may exist in any of those three contexts:

1. Tom believes that Mary wants to meet someone who is sailor and marry him.
2. Tom believes that there is a sailor and Mary wants to marry him.
3. There is a sailor, and Tom believes that Mary to marry wants him.

In Figure 10, two conceptual graphs represent the first and third sentences. In the CG on the left, the existential quantifier for the concept of the sailor is inside the situation that Mary wants. Whether the sailor actually exists and whether Tom or Mary knows his identity are not implied. The CG on the right explicitly states that the sailor exists. The nested contexts imply that Tom knows him and that Tom believes that Mary also knows him. The second option (not shown in Figure 10) would place the concept of the sailor inside the context of the proposition. It would not imply that he exists, but it would imply that Tom believes he exists and that Mary knows him.

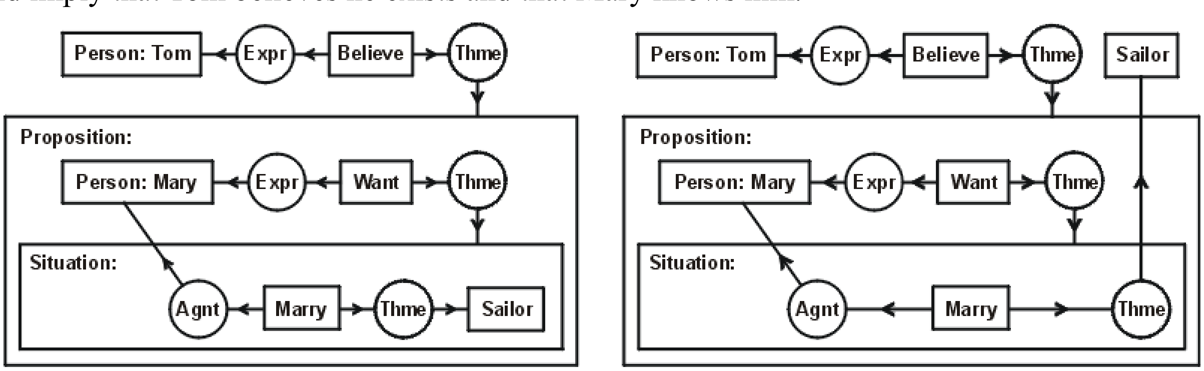

Figure 10: Two interpretations of *Tom believes that Mary wants to marry a sailor*

Metalevel reasoning with multiple kinds of modality is more general and flexible than Peirce's 1903 EGs with just two modal operators, possibility and necessity. (Dunn 1973; Hintikka 1976; Barwise & Perry 1983; Devlin 1991; McCarthy 1997; Hayes, Menzel, et al. 2006; Sowa 2010).

In summary, the EG framework of nested areas is fundamental. Over the years, Peirce also designed, developed, and suggested versions of metalanguage, higher-order logic, probability, modality, and virtual reality. To represent them, he discussed extensions of the EG notation. In L376 (December 1911), Peirce began to specify a new extension he called *Delta graphs*. Although his specification was incomplete, his applications could be represented by GEGs (Sowa, forthcoming).

4. Significs

In 1903, Peirce reviewed two books that were related to his lectures. Victoria Welby's book on significs contained examples and analyses related to his Harvard lectures in the spring. His comments on that book were detailed and enthusiastic. Bertrand Russell's book, however, covered issues that Peirce had developed twenty years earlier. Since his Lowell lectures in the fall went much further, his comments on Russell's book were short and lukewarm.

During the following decade, correspondence between Peirce and Welby strongly influenced both. In 1903, Peirce had adopted Kant's abstract phenomenology. But in 1904, he coined the new word *phaneroscopy*, which he discussed in terms that were closer to Welby's emphasis on observation and mental experience. In his letters to her, Peirce added examples that clarified the motivation and explained the details of his abstract analysis. His classification of the sciences in 1903 (Figure 1) illustrates the differences, Peirce had sharply distinguished mathematics, phaneroscopy, and the normative sciences. With her emphasis on examples, Welby showed how practical issues affected the details of each case. As a result of their correspondence, Peirce revised and generalized the foundation of his logic, semeiotic, and pragmatism.

Welby's writings, correspondence, and meetings also brought significs to the attention of philosophers, logicians, and linguists in Cambridge, Europe, and America (Petrilli 2020). Her diverse and prominent correspondents included Bertrand Russell, Lord Kelvin, H. G. Wells, and Nikola Tesla. At Whittier College in California, Albert Upton, a professor of English, discovered significs from discussions with I. A. Richards, who had become a professor at Harvard. As a result, Upton developed a one-year course named Significs. With the later name English 1 and 2, it became a required course for all first-year students. The course notes and exercises evolved into the book *Creative Analysis* (Upton & Samson 1961). From the back cover:

> On June 27, 1960, a front-page article in the New York Times announced the dramatic results of an experiment by Dr. Albert Upton, Chairman of the English Department at Whittier College, with a group of 280 freshmen. The students tested had worked with the exercises in *Creative Analysis* over a period of eight months. At the beginning of the period, they had been given a standard IQ test and scored an average of 109.5. At the end, they were given a second IQ test; their average score had been raised to 120 points, but some students had gains as large as 20 points, and one had gained 32 points.

Richard Samson, Upton's former student, teaching assistant, and co-author, wrote that article. He had organized Upton's course notes and exercises into book form. Charles Cooper, a colleague of Upton's who had spent a year as a visiting professor at Harvard, adopted related methods for his book, *The Arts and Humanity*. It included extended discussions and examples of esthetics, some of which became exercises in Upton's books.